p73

[1.1]

$$\vec{PO} = \vec{AB} \iff -\langle x, y\rangle = \langle 2, -1\rangle - \langle -3, 4\rangle$$

$$\iff \langle x, y\rangle = -\langle 5, -5\rangle$$

$$= \langle -5, 5\rangle$$

$\therefore\ P(-5, 5)$

[1.2]

$$\vec{AQ} = \frac{1}{2}\vec{AB} \iff \langle x, y\rangle - \langle -3, 4\rangle = \frac{1}{2}\langle 5, -5\rangle$$

$$\iff \langle x, y\rangle = \langle \tfrac{5}{2}, -\tfrac{5}{2}\rangle + \langle -3, 4\rangle$$

$$= \langle -\tfrac{1}{2}, \tfrac{3}{2}\rangle$$

$\therefore\ Q(-\frac{1}{2}, \frac{3}{2})$

[2] Given: $\vec{OA} = 2\vec{a}$, $\vec{OB} = 3\vec{b}$, $\vec{OC} = 6\vec{a} - 6\vec{b}$, $\vec{OD} = 6\vec{b} - 4\vec{a}$

[2.1] Prove A, B, C co-linear.

<u>Proof</u> :

We will show that $\vec{AB} = k\vec{AC}$.

$$\vec{AB} = \vec{OB} - \vec{OA} = 3\vec{b} - 2\vec{a}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = 6\vec{a} - 6\vec{b} - 2\vec{a} = 4\vec{a} - 6\vec{b}$$

Then $-2(-2\vec{a} + 3\vec{b}) = 4\vec{a} - 6\vec{b}$.

Since Both $\vec{AB}$ and $\vec{AC}$ go through Point A and $\vec{AC}$ is a scalar multiple of $\vec{AB}$, A, B, C lie on the same line.

[2.2] Prove $\vec{AB} \parallel \vec{OD}$

$$\vec{AB} = \vec{OB} - \vec{OA} = 3\vec{b} - 2\vec{a}$$

$$\vec{OD} = 6\vec{b} - 4\vec{a} = 2(3\vec{b} - 2\vec{a}) = 2\vec{AB}.$$

Since $\vec{OD} = 2\vec{AB}$, $\vec{OD} \parallel \vec{AB}$.

[3] Given $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$, $|\vec{a} + \vec{b}| = 1$

[3.1] $|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = 1$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \frac{1 - |\vec{a}|^2 - |\vec{b}|^2}{2}$$

$$= \frac{1-3-4}{2}$$

$$= \frac{-6}{2}$$

$$\therefore \vec{a} \cdot \vec{b} = -3$$

[3.2] $(\vec{a} - \vec{b}) \cdot (\vec{a} + 2\vec{b})$

$$= |\vec{a}|^2 - 2|\vec{b}|^2 + \vec{a} \cdot \vec{b}$$

$$= 3 - 2(4) - 3$$

JUST showed $\vec{a} \cdot \vec{b} = -3$

$$= -8$$

$$\therefore \vec{a} \cdot \vec{b} = -8$$

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[4] Given: $\vec{OP} = \langle 1,1 \rangle$, $\vec{OQ} = \langle 1-\sqrt{3}, 1+\sqrt{3} \rangle$

[4.1] Get angle made by $\vec{OP}$ and $\vec{OQ}$.

$$\cos\theta = \frac{\vec{OP} \cdot \vec{OQ}}{|\vec{OP}|\,|\vec{OQ}|}$$

$$= \frac{(1-\sqrt{3}) + (1+\sqrt{3})}{\sqrt{1+1}\,\sqrt{(1-\sqrt{3})^2 + (1+\sqrt{3})^2}}$$

$$= \frac{2}{\sqrt{2}\,\sqrt{1+3-2\sqrt{3} + 1+3+2\sqrt{3}}}$$

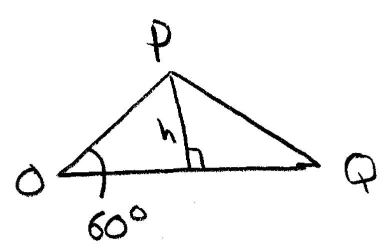
$$= \frac{2}{\sqrt{2}\,\sqrt{8}} = \frac{2}{4}$$

$$= \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\therefore \theta = 60°$$

[4.2] Area of ΔOPQ



You do not need correct shape of ΔOPQ, ONLY the hgt and BASE.

$$A_{\Delta OPQ} = \frac{1}{2} h b$$

$$= \frac{1}{2}\left(|\vec{OP}| \sin 60°\right)\left(|\vec{OQ}|\right)$$

$$= \frac{1}{2}\left[\sqrt{2} \cdot \frac{\sqrt{3}}{2}\right]\sqrt{8}$$

$$= \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{8}}{4}$$

$$\therefore A_{\Delta OPQ} = \sqrt{3} \text{ SQR units}$$

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[5] $l_1: \sqrt{3}x + y - 1 = 0$

$l_2: x + \sqrt{3}y + 2 = 0$, get angle between lines.

$\vec{n}_1 = \langle \sqrt{3}, 1 \rangle$

$\vec{n}_2 = \langle 1, \sqrt{3} \rangle$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$$= \frac{\sqrt{3} + \sqrt{3}}{\sqrt{4}\ \sqrt{4}}$$

$$= \frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$\therefore\ \theta = 30^\circ$

[6] Given: $|\vec{p}\cdot\vec{q}| \le |\vec{p}||\vec{q}|$. $\vec{p} = \langle a, b\rangle$, $\vec{q} = \langle x, y\rangle$

<u>Prove</u> $(ax+by)^2 \le (a^2+b^2)(x^2+y^2)$

<u>Proof</u>

$|\vec{p}\cdot\vec{q}| \le |\vec{p}||\vec{q}|$

$|\vec{p}\cdot\vec{q}|^2 \le |\vec{p}|^2|\vec{q}|^2$, since Both sides positive

$(ax+by)^2 = (\sqrt{a^2+b^2})^2(\sqrt{x^2+y^2})^2$

$\therefore$ $(ax+by)^2 = (a^2+b^2)(x^2+y^2)$

[7] P has velocity $\vec{v} = \langle 2, 5\rangle$.
At $t=0$, P at $A(-6,-2)$

[7.1] Get $\vec{p}(t)$

$\vec{p}(t) = \vec{a} + t\vec{v}$

$\therefore$ $\vec{p}(t) = \langle -6, -2\rangle + t\langle 2, 5\rangle$

or equivalently

$\vec{p}(t) = \langle 2t-6, 5t-2\rangle$

Either answer is O.K.

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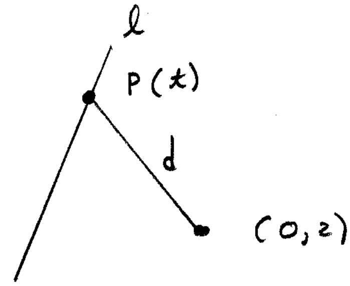
This solution includes discussion of how to think of it.

[7.2] We are asked to find the value of t for which distance of Line from point $(0,2)$ is minimal.



SOLN

Let P be any point on line.
Then $\vec{p}(t) = \langle 2t-6, 5t-2 \rangle$.
This means that at time t, the Point P has coordinates

$$P(2t-6, 5t-2)$$

The distance formula for two points yields

$$d = \sqrt{[(2t-6)-0]^2 + [(5t-2)-2]^2}$$

so $$d^2 = 4t^2 - 24t + 36 + 25t^2 - 40t + 16$$

$$d^2 = 29t^2 - 64t + 48$$

Since both sides positive, Finding t for which d^2 is minimum is equivalent to findind t for which d is minimum.

By completing the Square, we will find the t for which d^2 is minimum.

$$29t^2 - 64t + 48$$
$$= 29\left[t^2 - \frac{64}{29}t\right] + 48$$
$$= 29\left(t - \frac{64}{2\cdot 29}\right)^2 + \underbrace{48 - 29\left(\frac{64}{2\cdot 29}\right)}_{\text{positive}}$$

So d is minimum at $t = \frac{64}{2\cdot 29} = \frac{32}{29}$ sec

$\therefore$ $t = \frac{32}{29}$ sec

→

WITH OUT discussion, soln looks like this

$$d^2 = [(2t-6)-0]^2 + [(5t-2)-2]^2$$

$$= (2t-6)^2 + (5t-4)^2$$

$$= 4t^2 - 24t + 36 + 25t^2 - 40t + 16$$

$$= 29t^2 - 64t + 48$$

$$= 29\left[t^2 - \frac{64}{29}\right] + 48$$

$$= 29\left(t - \frac{64}{2\cdot 29}\right)^2 + 48 - 29\left(\frac{64}{2\cdot 29}\right)$$

$\therefore$ d is minimum when $t = \frac{32}{29}$